Straight line linkages
Straight line linkages

Generation of straight line motion using linkage mechanisms has always been a common requirement in machine design practice. Although exact straight line cannot be generated using simple mechanisms though some simple mechanisms are designed such that they can produce approximate straight lines for short range of motion. These approximate straight line mechanisms has important applications in machine design. These mechanisms were used extensively in classical machines such as steam engines. Perfect straight lines can also be generated using linkage mechanisms but those are relatively complex mechanisms.
Straight line linkages

There are two classes of straight line mechanisms

- **Approximate straight line linkages**
  - Watt's linkage
  - Hoekens linkage
  - Chebyshev linkage
  - Roberts Mechanism

- **Exact straight line linkages**
  - Sarrus linkage
  - Peaucellier–Lipkin linkage
  - Hart's A-frame
Straight line linkages

Watt's linkage

Roberts mechanisms

Chebyshev linkage

Peaucellier–Lipkin linkage

Source: Eugene S. Ferguson: KINEMATICS OF MECHANISMS FROM THE TIME OF WATT (1962)
A. B. Kempe, HOW TO DRAW A STRAIGHT LINE (1877)
Straight line linkages

Watt's linkage

Roberts mechanisms

Chebyshev linkage

Peaucellier–Lipkin linkage

Source: lecture Massachusetts Institute of Technology (2010)
Watt's linkage / Watt's Parallel Motion

Watt's linkage (also known as the parallel linkage) is a type of mechanical linkage invented by James Watt in which the central moving point of the linkage is constrained to travel on an approximation to a straight line. It was described in Watt's patent specification of 1784 for the Watt steam engine. It is also used in automobile suspensions, allowing the axle of a vehicle to travel vertically while preventing sideways motion.

Hand-drawn diagram by James Watt (1808) in a letter to his son, describing how he arrived at the design.
**Watt's linkage** / Watt's Parallel Motion

Watt's linkage is a simple four bar mechanism of double-rocker type with the two rockers connected through a coupler. When the two rockers move the mid-point of the coupler moves in an almost straight line path for the motion close to coupler's mean position. If something is hinged to the middle point of the coupler of Watt's linkage it will be constrained to move in straight line path close to the coupler's mean position.

Link ratios are: $L_1 = L_3$.
Point P is in the middle of L2
This property of Watt's linkage is utilized in construction of rear axle suspension system of car to prevent sideways motion of car body relative to the rear axle.

It consists of two horizontal rods of equal length mounted at each side of the chassis. In between these two rods, a short vertical bar is connected. The center of this short vertical rod – the point which is constrained in a straight line motion - is mounted to the center of the axle. All pivoting points are free to rotate in a vertical plane.

The linkage can be inverted, in which case the centre P is attached to the body, and L1 and L3 mount to the axle. This reduces the unsprung mass and changes the kinematics slightly. This is used on Australian V8 Supercars.
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Watt's linkage

Whiteline Watt's linkage
Ford Mustang GT, Shelby GT
Watt's linkage can also be used to prevent axle movement in the longitudinal direction of the train. This application involves two Watt's linkages on each side of the axle, mounted parallel to the driving direction, but just a single 4-bar linkage is more common in racing suspension systems.
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Chebyshev linkage

The Chebyshev linkage is a mechanical linkage that converts rotational motion to approximate straight-line motion.

It was invented by the nineteenth-century mathematician Pafnuty Chebyshev, who studied theoretical problems in kinematic mechanisms. One of the problems was the construction of a linkage that converts a rotary motion into an approximate straight-line motion.

The straight-line linkage confines the point $P$ – the midpoint on the link $L_3$ – on a straight line at the two extremes and at the center of travel. ($L_1$, $L_2$, $L_3$, and $L_4$ are as shown in the illustration.) Between those points, point $P$ deviates slightly from a perfect straight line. The proportions between the links are

$$L_1 : L_2 : L_3 = 2 : 2.5 : 1 = 4 : 5 : 2.$$
The Hoekens linkage is a four-bar mechanism that converts rotational motion to approximate straight-line motion with approximate constant velocity. The precise design trades off straightness, lack of acceleration, and what proportion of the driving rotation is spent in the linear portion of the full curve. The Hoekens linkage is a cognate linkage of the Chebyshev linkage.
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Robert's Straight Line Mechanism

Like the Chebyshev's mechanism Robert's approximate straight line mechanism is a symmetrical four bar linkage. The construction of Robert's mechanism is different from the approximate straight line mechanisms discussed so far, in the sense that, this mechanism has an extension to the coupler at the coupler mid point. This extension is perpendicular to the line joining the two adjacent joints. The end point of the coupler extension generates an approximate straight line for the motion between the fixed pivots.

This approximate straight line mechanism is generally used for linear guidance of the tracing point. The point required to traverse on straight line is constrained to the end point of the coupler extension. Robert's straight line mechanism is normally used in the coupler driven mode, that is, the mechanism is not driven by either of the cranks or rockers instead the coupler extension is used to just guide the requisite point along an approximate straight line.
the Roberts had proposed another approximate solution, based on a four-bar mechanism and on a BPC blade in the shape of an isosceles triangle. In this case one must have

\[ AB = BP = PC = CD \]

and

\[ AD = 2 \cdot BC, \]

the vertex P of the blade traces for a remarkably long tract a nearly straight line.
The Peaucellier–Lipkin linkage (or Peaucellier–Lipkin cell, or Peaucellier–Lipkin Inversor), invented in 1864, was the first planar straight line mechanism -- the first planar linkage capable of transforming rotary motion into perfect straight-line motion. It is named after Charles-Nicolas Peaucellier (1832–1913), a French army officer, and Yom Tov Lipman Lipkin. Until this invention, no planar method existed of producing straight motion without reference guideways, making the linkage especially important as a machine component and for manufacturing. In particular, a piston head needs to keep a good seal with the shaft in order to retain the driving (or driven) medium.

The mathematics of the Peaucellier–Lipkin linkage is directly related to the inversion of a circle.
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Peaucellier–Lipkin linkage

In the geometric diagram of the apparatus, six bars of fixed length can be seen: OA, OC, AB, BC, CD, DA. The length of OA is equal to the length of OC, and the lengths of AB, BC, CD, and DA are all equal forming a rhombus. Also, point O is fixed. Then, if point B is constrained to move along a circle (shown in red) which passes through O, then point D will necessarily have to move along a straight line (shown in blue). On the other hand, if point B were constrained to move along a line (not passing through O), then point D would necessarily have to move along a circle (passing through O).
The complexity of the mechanisms to generate exact straight lines can be reduced by introduction of one or more slider crank linkages. It is possible to generate an exact straight line using the slider crank mechanism but the range of motion is limited. One such example is Scott-Russell Mechanism as shown in the figure. Based on the geometry of the linkage the output motion is a simple sine function of the drive link or a simple harmonic motion. It is evident from the figure that this mechanism is made up of isosceles triangles, \( \text{AB, AC and AO}_2 \) are of equal lengths.
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Source: Robert L. Norton, DESIGN OF MACHINERY (AN INTRODUCTION TO THE SYNTHESIS AND ANALYSIS OF MECHANISMS AND MACHINES)
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