CREATING OF KINEMATIC CHAINS
Mechanism

Definitions
1. a system or structure of moving parts that performs some function
2. is each system reciprocally joined moveable bodies
   • the bodies we call members or link (they can be rigid, elastic, flexible etc.)
Degrees of Freedom

• The system's DOF is equal to the number of independent parameters (measurements) which are needed to uniquely define its position in space at any instant of time.
• A rigid body in plane motion has three DOF.
• Any rigid body in three-space has six degrees of freedom
Types of motions

A rigid body free to move within a reference frame will, in the general case, have complex motion, which is a simultaneous combination of rotation and translation.

In three-dimensional space, there may be rotation about any axis and also simultaneous translation which can be resolved into components along three axes.

In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane.

We will limit our present discussions to the case of planar (2-D) kinematic systems.
Types of motions

**Pure rotation:** the body possesses one point (center of rotation) which has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

**Pure translation:** all points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation.

**Complex motion:** a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.
Links, Joints and Kinematic Chains

A **link** is an (assumed) rigid body which possesses at least two nodes which are points for attachment to other links.

- **Binary link**: A link which is connected to two other links is called a binary link.
- **Ternary link**: A link which is connected to three other links is called a ternary link.
- **Quaternary link**: A link which is connected to four other links is called a quaternary link.
Links, Joints and Kinematic Chains

Some of the common types of links are:

- Binary link - one body with two nodes.
- Ternary link - one body with three nodes.
- Quaternary link - one body with four nodes.
- Pentagonals – one body with five nodes.
- Hexagonals – one body with six nodes.
A joint is a connection between two or more links (at their nodes), which allows some motion (or potential motion) between the connected links. Joints (also called kinematic pairs) can be classified in several ways:

1. By the number of degrees of freedom allowed at the joint.
2. By the type of contact between the elements (line, point, or surface).
3. By the type of physical closure of the joint: either force or form closed.
4. By the number of links joined (order of the joint).
1. Classification by the Number of Degrees of Freedom allowed at the joint

We can classify Joints by the number of degrees of freedom allowed at the joint as:

- One-Freedom Joints or Full Joints - examples of one freedom joints are a rotating pin joint (R) or a translating slider Joint (P).
- Two Freedom Joints or Half Joints - examples of two freedom joints are link against plane or a pin in slot (C).
- Three Freedom Joints - examples of three freedom joints are a spherical (G) or ball-and-socket joints.
Links, Joints and Kinematic Chains

1. Classification by the Number of Degrees of Freedom allowed at the joint

R  Revolute pair  \( i=1 \)  (a)
P  Prismatic pair  \( i=1 \)  (b)
S  Screw pair  \( i=1 \)  (c)

a) Relative movement is rotational movement about an axis defined by the coordinate \( \psi \).
b) Relative movement is movement in the axial direction of the pin defined by the coordinates \( s \).
c) Relative movement of the screw is defined by the coordinate \( \psi \) rotational movement around the screw axis or coordinate \( s \) with the sliding movement in the axial direction of the screw. Coordinates \( \psi, h \) are bonded \( \Delta s - h \frac{\Delta \psi}{2\pi} \) where \( h \) is the stroke of the bolt in one revolution..
Links, Joints and Kinematic Chains

1. Classification by the Number of Degrees of Freedom allowed at the joint

<table>
<thead>
<tr>
<th></th>
<th>Spheric pair</th>
<th>i=3</th>
<th>(d)</th>
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<tbody>
<tr>
<td>G</td>
<td>Cylinder pair</td>
<td>i=2</td>
<td>(a)</td>
</tr>
<tr>
<td>C</td>
<td>Planar pair</td>
<td>i=3</td>
<td>(f)</td>
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</tbody>
</table>

**d)** Relative motion is defined by a spherical three coordinates: two coordinates $\alpha$, $\varphi$ specify the direction of the axis of rotation and are $\psi$ of rotation about the axis.

**e)** Relative movement is a combination of rotational movement about the axis of the cylinder and sliding along the cylinder axis; corresponding coordinates are $\psi$, $s$.

**f)** Relative movement is composed of two displacements $x$, $y$, $\psi$ of rotation about an axis perpendicular to the plane of movement.
2. Classification by the Type of Contact between elements

We can classify Joints by the type of contact as Lower Pairs or Higher Pairs.

- If joints have surface contact, they are called Lower pair (as with a pin surrounded by a hole).
- If joints have point or line contact, they are called Higher pair.

The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces.
Links, Joints and Kinematic Chains

2. Classification by the Type of Contact between elements

The six possible lower pairs are:

- Revolute (R),
- Prismatic (P),
- Screw/Helical (H),
- Cylindric (C),
- Spherical (G),
- Flat (F).
3. Classification by the Type of Physical Closure of the Joint

We can classify Joints by the type of physical closure of the joint as Force closed or Form closed.

- A form closed joint is kept together or closed by its geometry. A pin in a hole or a slider in a two-sided slot are form closed.

- In contrast, a force-closed joint, such as a pin in a half-bearing or a slider on a surface, requires some external force to keep it together or closed. This force could be supplied by gravity, a spring, or any external means.
Links, Joints and Kinematic Chains

4. Classification by the Order of Joints

- We can classify Joints by the order of joints as 1st order, 2nd order and so on.
- Order is defined as the number of links joined minus one.
- It takes two links to make a single joint; thus the simplest joint combination of two links has order one.
- Joint order has significance in the proper determination of overall degree of freedom for the assembly.
Links, Joints and Kinematic Chains

Kinematic chain:
It is a combination of several successively arranged joints constituting a complex motion system.
The kinematic chain may be:
- open or closed
- simple or composite
- Free or fixed
- Planar or spatial (trajectories are planar or spatial curves)
The kinematic chain may be open or closed.

- In a closed kinematic chain, the distal segment is fixed and the end segments are unite to form a ring or a circuit. When one link moves all the other links will move in a predictable pattern.

- In an open kinematic chain, the distal segment terminates free in space.
The kinematic chain may be simple or composite.
(composite – one member joints with three and more kinematic pairs)

The kinematic chain may be free or fixed.
PLANAR KINEMATIC CHAINS
Planar Kinematic Chain

(a) Closed
   Composite
   free

(b) Open
   Simple
   free

(c) Combine (open, close)
   Composite
   free

(d) Open
   Simple
   fixed
Planar Kinematic Chain

A first question is “the degree of freedom of a planar kinematic chain”. Planar element without any connection has three independently possibilities to move (two translations and a rotation) and that any revolute joint takes away two translation possibilities.

Grübler formulated the degree of freedom of the basic chain as:

\[ i = 3n - 2j \]

- \( i \) – degrees of freedom of a planar kinematic chain
- \( n \) – number of members/links of kinematic chain (without frame)
- \( j \) – number of kinematic pairs

Grubler’s equation/Count
Planar Kinematic Chain

\[ i = 3n - 2j \]

- **i** – Degrees of Freedom
- **n** – number of members of kinematic chain (without frame)
- **j** – number of kinematic pairs

To avoid misinterpretation of this formula it must be noted that:

- a rigid sub-structure must be considered as one element (e.g. a triangle consisting of three binary links must be regarded as one ternary link),
- a point in the drawing may not be precisely the same as a single revolute joint:
  - a loose end of a link or a coupler point is not a revolute joint, and
  - a double joint, should be regarded “split up”. Consequently this point includes two single revolute joints, while one of the three elements holds the two connections and should be regarded as a ternary link instead of a binary link.

Grubler’s equation/Count
Planar Kinematic Chain

Example

\[ i = 3n - 2j \]

- \( i \) – degrees of freedom of a planar kinematic chain
- \( n \) – number of members/links of kinematic chain (without frame)
- \( j \) – number of kinematic pairs
Planar Kinematic Chain

\[ i = 3n - 2j \]

- \( i \) – Degrees of Freedom
- \( n \) – number of members of kinematic chain (without frame)
- \( j \) – number of kinematic pairs
Planar Kinematic Chain

How many degrees of freedom has the following kinematic chain?
Planar Kinematic Chain

How many degrees of freedom has the following kinematic chain?

\[ i = 3n - 2j \]

- \( i \) – Degrees of Freedom
- \( n \) – number of members of kinematic chain (without frame)
- \( j \) – number of kinematic pairs

\[ n = 13, \ j = 19 \]

\[ i = 3.13 - 2.19 = 1 \]
Kinematic Chain with constrained motion

A type of kinematic chain is one with constrained motion, which means that a definite motion of any link produces unique motion of all other links. Thus motion of any point on one link defines the relative position of any point on any other link.

Kinematic Chain with constrained motion has one degree of freedom (fixed) or four degree of freedom (free).

\[ i = 3n - 2j = 4 \]
\[ j = (3n - 4)/2 \]
Kinematic Chain with constrained motion

\[ n = n_2 + n_3 + n_4 + \ldots \ldots + n_i = \sum_{i=2}^{i} n_i \]

the number of single revolute joints follows, instead by counting from:

\[ e = 2j = 2n_2 + 3n_3 + 4n_4 + \ldots \ldots + in_i = \sum_{i=2}^{i} in_i \]

Closed kinematic chain is characterised by the fact that all links have minimal two (revolute) joints with another link. Without further prove it is stated that, for a closed kinematic chain, the value of \( i_{\text{max}} \) (the highest amount of single revolute joints that can possibly be at one element) is determined by:

\[ 2 \leq i_{\text{max}} \leq \frac{n}{2} \]

In case of an odd number \( n \) this value should be truncated to the nearest lower integer value.
Kinematic Chain – Four members

\[
n = n_2 + n_3 + n_4 + \ldots + n_i = \sum_{i=2}^i n_i \tag{1}
\]

\[
e = 2j = 2n_2 + 3n_3 + 4n_4 + \ldots + in_i = \sum_{i=2}^i in_i \tag{2}
\]

\[
i_{max} = \frac{n}{2} = 2 \tag{3}
\]

We can start with \(n=4\) elements forming a closed kinematic chain. The value of the highest amount of single revolute joints that can possibly be at one element \((i_{max})\) is \(=2\), which means that only binary links are applicable. Equation (2) shows that this chain has 4 revolute joints. With 4 bars there is obviously only one way to make a closed chain, like drawn in figure.

\[
n_2 = 4 \tag{1}
\]

\[
2n_2 = 2j=8 \tag{2}
\]

\[
j = (3n-4)/2=4
\]

\(j – \text{number of kinematic pairs}\)
Kinematic Chain – Six members

\[ n = n_2 + n_3 + n_4 + \ldots + n_i = \sum_{i=2}^{i} n_i \]  \hspace{1cm} (1)

\[ e = 2j = 2n_2 + 3n_3 + 4n_4 + \ldots + in_i = \sum_{i=2}^{i} in_i \] \hspace{1cm} (2)

\[ i_{\text{max}} = \frac{n}{2} = 3 \] \hspace{1cm} (3)

With \( n=6 \) elements there may be both binary and ternary links according equation (3). The structure relations provide equations for their amount. From equation (3) it follows that this chain has 7 revolute joints.

\[ n_2 + n_3 = 6 \] \hspace{1cm} (1)

\[ 2n_2 + 3n_3 = 2j = 14 \] \hspace{1cm} (2)

\[ j = (3n-4)/2 = 7 \]

\( j \) – number of kinematic pairs
Kinematic Chain — Six members

\[ n = n_2 + n_3 + n_4 + \ldots + n_i = \sum_{i=2}^{\infty} n_i \]  \hspace{1cm} (1)

\[ e = 2j = 2n_2 + 3n_3 + 4n_4 + \ldots + \sum_{i=2}^{\infty} in_i \]  \hspace{1cm} (2)

\[ i_{\text{max}} = \frac{n}{2} = 3 \]  \hspace{1cm} (3)

The six-bar closed chain has one specification (four binary links and two ternary links). By systematic variation it can be found that two different configurations can be made. They are well known as:

\[ a) \text{ Chain of Watt} \]
\[ \text{ (the two ternary elements are connected by a joint)}, \]

\[ b) \text{ Chain of Stephenson} \]
\[ \text{ (no connection between the two ternary links)}. \]

\[ n_2 + n_3 = 6 \]  \hspace{1cm} (1)

\[ 2n_2 + 3n_3 = 2j = 14 \]  \hspace{1cm} (2)

\[ n_2 = 4, \ n_3 = 2. \]
Kinematic Chain – Six members

Watt’s and Stephenson’s chain can have different choices for their fixed link. In literature they are known as Watt-1,2 and Stephenson-1,2,3 mechanism.

In principle all these mechanisms have topological variants when one or more revolute joints will be replaced by a sliding pair.

An interesting practical application is the Watt-2 mechanism that can be considered as a series connection of two four-bar mechanisms.

Different six-bar mechanisms by fixed link choice
For n=8 elements the following can be derived. From equation (3): $i_{\text{max}} = 4$, so the kinematic chains may contain binary, ternary and quaternary links. From equation (2): the number of revolute joints is 10.

two equations have three unknowns ($n_2$, $n_3$ and $n_4$), so there is more than one solution possible.
\[ i_{\text{max}} = \frac{n}{2} = 4 \]

**Kinematic Chain – Eight members**

\[
\begin{align*}
  n_2 + n_3 + n_4 &= n \\
  2n_2 + 3n_3 + 4n_4 &= 2j \\
  n_2 + n_3 + n_4 &= 8 \\
  2n_2 + 3n_3 + 4n_4 &= 20 \\
  n_3 + n_4 &= 8 - n_2 \\
  3n_3 + 4n_4 &= 20 - 2n_2
\end{align*}
\]

\[ n_3 = \begin{vmatrix} 8 - n_2 & 1 \\ 20 - 2n_2 & 4 \end{vmatrix} = 12 - 2n_2 \]

\[ n_4 = \begin{vmatrix} 1 & 8 - n_2 \\ 3 & 20 - n_2 \end{vmatrix} = -4 + n_2 \]

\[ 6 \geq n_2 \geq 4 \]

<table>
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<th>( n_4 )</th>
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<td>2</td>
</tr>
<tr>
<td>III.GROUP</td>
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</tr>
</tbody>
</table>

16 different configurations can be made

\[ n_3, n_4 – \text{integral positive numbers.} \]
Kinematic Chain – Eight members

<table>
<thead>
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<th>$n_3$</th>
<th>$n_4$</th>
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<tbody>
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I.GROUP 9 variants
Kinematic Chain – Eight members

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<tbody>
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<td>1</td>
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<tr>
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II.GROUP 5 variants
Kinematic Chain – Eight members

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<td>1</td>
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<tr>
<td>6</td>
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</tbody>
</table>

III.GROUP 2 variants
Kinematic Chain – Ten members

\[ n = n_2 + n_3 + n_4 + \ldots \ldots + n_i = \sum_{2}^{i} n_i \]  \hspace{1cm} (1)

\[ e = 2j = 2n_2 + 3n_3 + 4n_4 + \ldots \ldots + in_i = \sum_{2}^{i} in_i \]  \hspace{1cm} (2)

\[ i_{max} = \frac{n}{2} = 5 \]  \hspace{1cm} (3)

For \( n = 10 \) elements the following can be derived. From equation (3): \( i_{max} = 5 \), so the kinematic chains may contain binary, ternary, quaternary and pentagonals links. From equation (2): the number of revolute joints is 13.

\[ n_2 + n_3 + n_4 + n_5 = 10 \]  \hspace{1cm} (1)

\[ 2n_2 + 3n_3 + 4n_4 + 5n_5 = 26 \]  \hspace{1cm} (2)

\[ j = \frac{(3n-4)}{2} = 13 \]

\( j \) – number of kinematic pairs

two equations have four unknowns \( (n_2, n_3, n_4 \) and \( n_5 \)), so there is more than one solution possible.
Kinematic Chain – Ten members

\[ i_{\text{max}} = \frac{n}{2} = 5 \]

\[ n_2 + n_3 + n_4 + n_5 = 10 \]
\[ 2n_2 + 3n_3 + 4n_4 + 5n_5 = 26 \]

\[ n_2 + n_3 = 10 - n_4 - n_5 \]
\[ 2n_2 + 3n_3 = 26 - 4n_4 - 5n_5 \]

\[ n_2 = \begin{vmatrix} 10 - n_4 - n_5 \\
26 - 4n_4 - 5n_5 
\end{vmatrix} = 4 + n_4 + 2n_5 \]

\[ n_3 = \begin{vmatrix} 1 & 10 - n_4 - n_5 \\
2 & 26 - 4n_4 - 5n_5 \end{vmatrix} = 6 - 2n_4 - 3n_5 \]

\[ 2n_4 + 3n_5 \leq 6 \]

The kinematic chain with 10 members has 7 specifications and 230 different chain configurations.

Comment: the kinematic chain with 12 members has 15 specifications and 6856 different configurations.
Kinematic Chain – Ten members

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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</table>
Kinematic Chain – Ten members
SPATIAL KINEMATIC CHAINS
Spatial Kinematic Chain

A system of $n$ rigid bodies moving in space has $6n$ degrees of freedom. Kinematic pairs (joints) that connect bodies in the system remove degrees of freedom and reduce mobility. The result is that the mobility of a system formed from $n$ moving links and $d$ kinematic pairs each with freedom $i_j$, $j = 1, \ldots, d$, is given by

$$I = 6(n - d - 1) + \sum_{1}^{d} i_j$$

$I$ – degrees of freedom of a spatial kinematic chain
$n$ – number of members/links of kinematic chain (with frame)
$d$ – number of kinematic pairs
$i_j$ – degrees of freedom of $j$-kinematic pairs
Spatial Kinematic Chain

- **R** Revolute pair \( i=1 \) (a)
- **P** Prismatic pair \( i=1 \) (b)
- **S** Screw pair \( i=1 \) (c)

- **G** Spheric pair \( i=3 \) (d)
- **C** Cylinder pair \( i=2 \) (a)
- **F** Planar pair \( i=3 \) (f)
Spatial Kinematic Chain

Spatial four-bar linkage with revolute kinematic pairs only – wrong option
Spatial Kinematic Chain

\[ I = 6(n - d - 1) + \sum_{1}^{d} i_j \]

\[ I = 6(4 - 4 - 1) + (1 + 3 + 1 + 2) = 1 \]

Spatial four-bar linkage – good option
Spatial Kinematic Chain

\[ I = 6(n - d - 1) + \sum_{i=1}^{d} i_j \]

\[ I = 6(4 - 4 - 1) + (1 + 2 + 3 + 1) = 1 \]

Spatial crank mechanism
Spatial Kinematic Chain

\[ I = 6(n-d-1) + \sum_{1}^{d} i_j \]

\[ I = 6(4-4-1)+(1+2+1+3) = 1 \]
Spatial Kinematic Chain

\[ I = 6(n - d - 1) + \sum_{1}^{d} i_j \]

\[ I = 6(4-4-1) + (1+3+3+1) = 2 \]
A spherical mechanism is a mechanical system in which the bodies move in a way that the trajectories of points in the system lie on concentric spheres. The rotational axes of hinged joints that connect the bodies in the system pass through the center of these circle
Spherical mechanism

Special type of spherical mechanism is a universal joint.
A universal joint, (universal coupling, U-joint, Cardan joint, Spicer or Hardy Spicer joint, or Hooke's joint) is a joint or coupling in a rigid rod that allows the rod to 'bend' in any direction, and is commonly used in shafts that transmit rotary motion. It consists of a pair of hinges located close together, oriented at 90° to each other, connected by a cross shaft. The universal joint is not a constant-velocity joint.

Angular (rotational) output shaft speed $\omega_2$, versus rotation angle $\gamma_1$, for different bend angles $\beta$, of the joint

Output shaft rotation angle, $\gamma_2$, versus input shaft rotation angle, $\gamma_1$, for different bend angles, $\beta$, of the joint
Kinematic Chain

Literature:


https://en.wikipedia.org/wiki/Linkage_%28mechanical%29